Baulkham Hills High School

2023 Year 12 Trial Examination

Mathematics Advanced

General Instructions

- Reading time - 10 minutes
- Working time - 3 hours
- Write using black pen.
- Calculators approved by NESA may be used.
- A reference sheet is provided at the back of this paper.
- In Questions 11-33, show relevant mathematical reasoning and/or calculations.

Total marks: 100

Section I - 10 marks (pages 4 - 8)

- Attempt Questions 1-10
- Allow about 15 minutes for this section.

Section II - 90 marks (pages 9 - 31)

- Attempt Questions 11 - 33
- Allow about 2 hours and 45 minutes for this section.

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

1 Stephanie bakes cupcakes to sell. The cost C, in dollars, for x cupcakes to be produced is given by the equation $C = 200 + 2x$. Each cupcake is then sold for \$4. How many cupcakes does Stephanie need to make to break even?

A. 0

B. 25

C. 100

D. 200

2 What is $\dfrac{4\sqrt{2} \times 5\sqrt{34}}{\sqrt{2}+1}$ fully simplified?

A. $\dfrac{20\sqrt{68}}{\left(\sqrt{2}+1\right)}$

B. $20\sqrt{68}$

C. $40\sqrt{34} - 40\sqrt{17}$

D. $40\sqrt{34} + 40\sqrt{17}$

3 The number of heads tossed, X, when 3 coins are tossed can be shown in the discrete probability distribution below:

x	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Which of the following is the expected value of X?

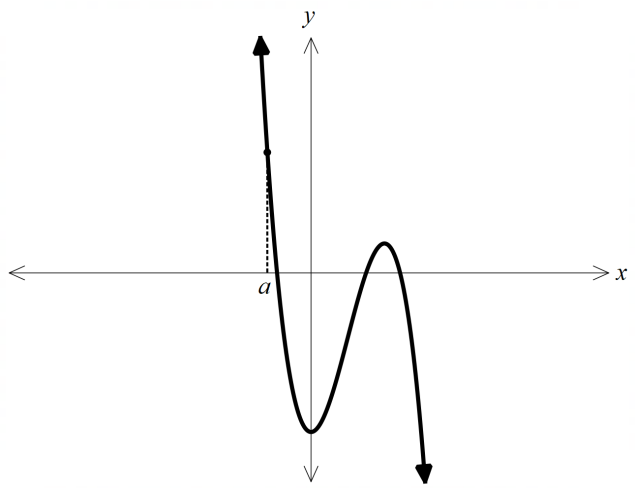
A. 1.5

B. 0.866 (3 decimal places)

C. 0.75

D. 2.25

4 The diagram below shows the graph of the function $f(x)$.



Which of the following is true about $f'(a)$ and $f''(a)$?

A. $f'(a) > 0$ and $f''(a) > 0$

B. $f'(a) > 0$ and $f''(a) < 0$

C. $f'(a) < 0$ and $f''(a) > 0$

D. $f'(a) < 0$ and $f''(a) < 0$

5 Which of the following could describe a probability density function?

A. $f(x) = \begin{cases} \sin 2x & 0 \le x \le \pi \\ 0 & \text{otherwise} \end{cases}$

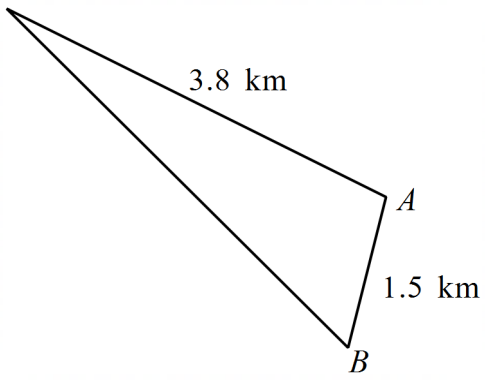
B. $f(x) = \begin{cases} 5 & 0 \le x \le 20 \\ 0 & \text{otherwise} \end{cases}$

C. $f(x) = \begin{cases} 1-x & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$

D. $f(x) = \begin{cases} 1.2 + 0.125x & 0 \le x \le 0.8 \\ 0 & \text{otherwise} \end{cases}$

6 Judy is participating in an orienteering competition. She runs 3.8 kilometres on a bearing of 140° to Checkpoint A, then turns on a bearing of 200° and runs 1.5 kilometres to Checkpoint B. Judy then returns to her starting point.

Some of this information is shown in the diagram below.



How far does Judy need to run to return to the starting point to 2 decimal places?

A. 2.02 km
B. 2.33 km
C. 4.73 km
D. 22.39 km

7 The distribution of the amount of dirt (in tons) sold by a landscaping company in a given week is a continuous random variable X, with probability distribution function:

$$f(x) = \begin{cases} 3(1-x)^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

What is the expected value of the distribution (to 2 decimal places)?

A. 0.04
B. 0.19
C. 0.25
D. 0.50

8 Which of the following is the domain and range of $y = \dfrac{x^2+1}{x^2-1}$?

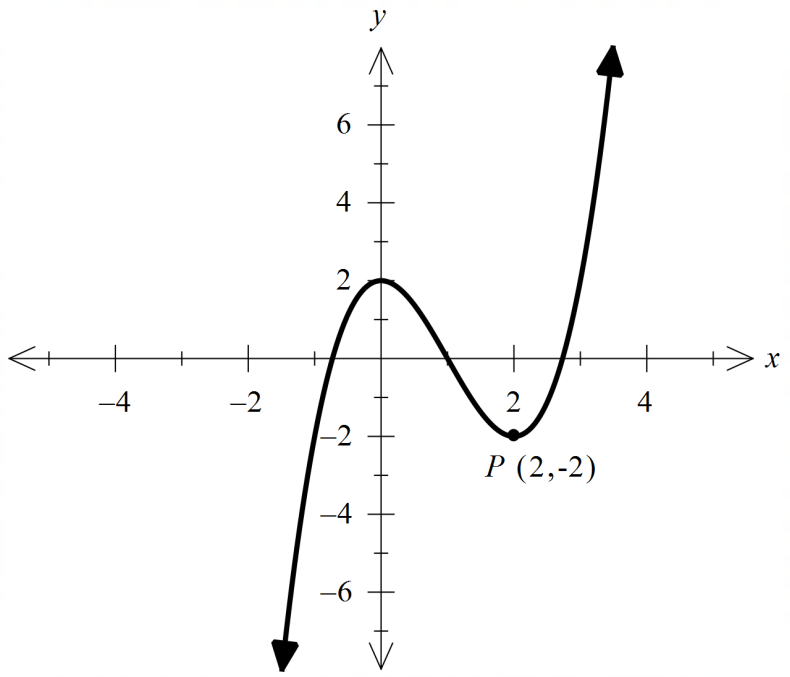
A. Domain: $x \in (-\infty,-1) \cup (-1,1) \cup (1,\infty)$ Range: $y \in (-\infty,-1) \cup (1,\infty)$

B. Domain: $x \in (-\infty,-1) \cup (-1,1) \cup (1,\infty)$ Range: $y \in (0,\infty)$

C. Domain: $x \in (-\infty,-1) \cap (-1,1) \cap (1,\infty)$ Range: $y \in (-\infty,-1) \cap (1,\infty)$

D. Domain: $x \in (-\infty,-1) \cap (-1,1) \cap (1,\infty)$ Range: $y \in (0,\infty)$

9 P lies on the graph of $y = f(x)$ as shown on the diagram below. A transformation maps the graph of $f(x)$ to $g(x)$ such that $g(x) = 2f(1-2x)$. The same transformation maps the point P to P'.



What are the coordinates of P'?

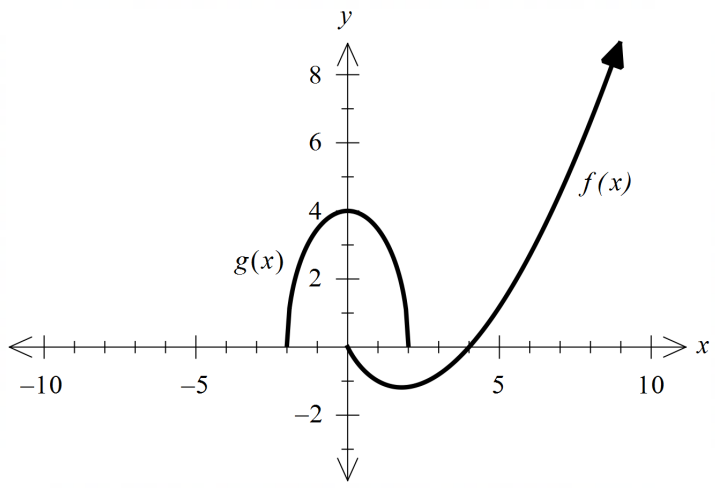
A. $(2,-6)$

B. $\left(\dfrac{-1}{2},-4\right)$

C. $\left(\dfrac{1}{2},-4\right)$

D. $(-2, -4)$

10 The diagram below shows the graph of the functions $f(x)$ and $g(x)$.



What is the domain for which $f(g(x))$ is defined?

A. $x \geq -2$

B. $x \geq 0$

C. $0 \leq x \leq 2$

D. $-2 \leq x \leq 2$

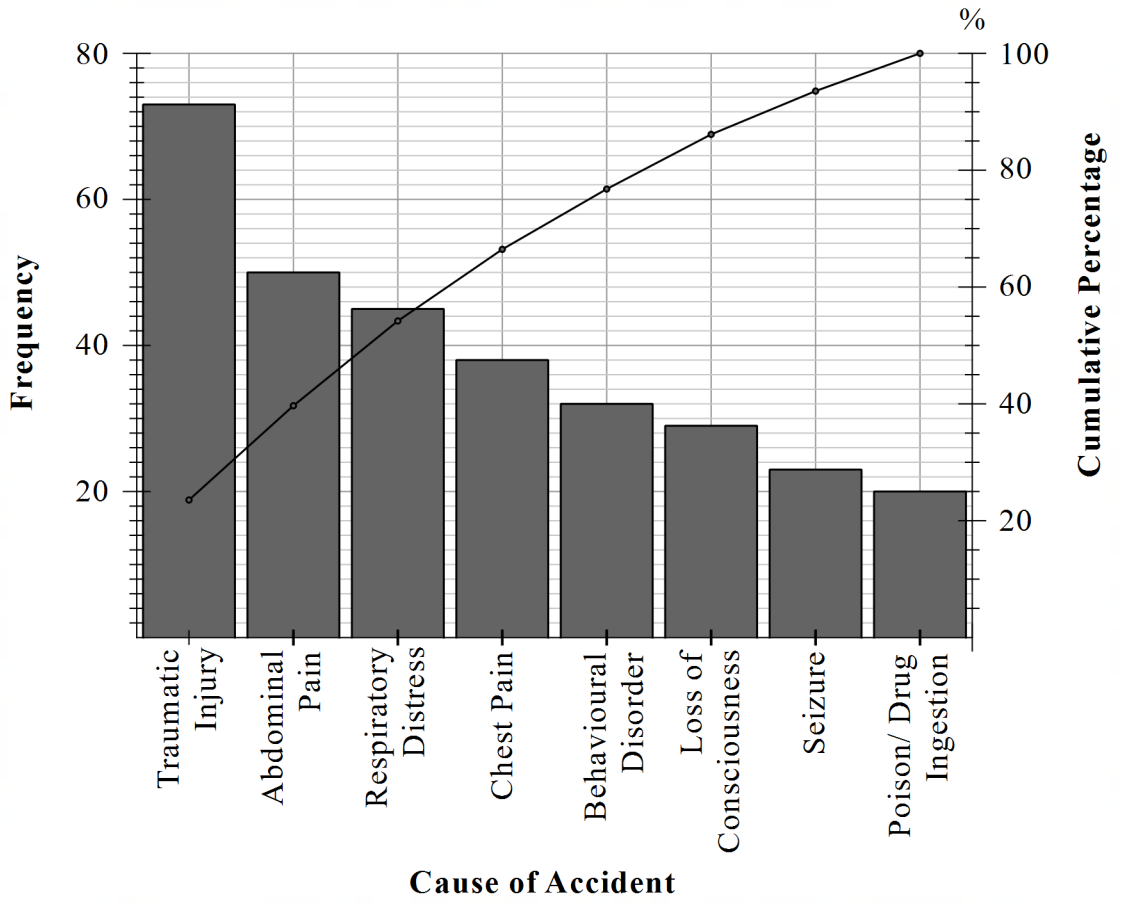
End of Section I

Section II

90 marks
Attempt Questions 11-33
Allow about 2 hours and 45 minutes for this section
Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response. Your responses should include relevant mathematical reasoning.

Question 11 (1 mark)

The reasons for an ambulance call out for a region are shown in the Pareto Chart below.



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Approximately what percentage of ambulance call outs were to attend a patient who had Respiratory Distress?

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Question 12 (7 marks)

Differentiate each of the following:

a) $y = 5x^4 - 14x^2 + 3$. **1**

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b) $y = (8 - 5x)^4$. **2**

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c) $y = x^2 \ln(3x + 5)$. **2**

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d) $y = \dfrac{e^x}{10x - 1}$. **2**

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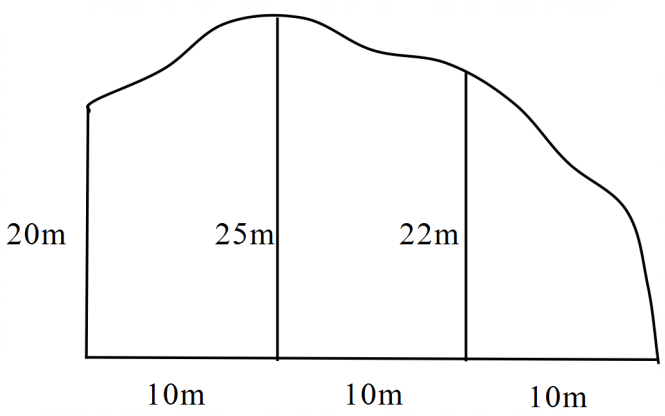
Question 13 (3 marks)

Shasrica purchases a block of land as shown in the diagram.



a) Using three applications of the Trapezoidal Rule, find the area (A) of the block of land. **2**

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b) Explain whether the area found by the Trapezoidal Rule would be an over estimate or under estimate. **1**

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Question 14 (2 marks)

A function which passes through the point $(2,-1)$, has gradient function $f'(x) = 8x^3 - 3x^2$. **2**
Find the equation of the function.

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Question 15 (4 marks)
A sapling with a height of 50cm is planted in the ground. After 1 week of being planted, it grows 20% of its height. Each week thereafter, it grows 20% of the previous week's growth.

a) How much will it grow in the 5^{th} week? **2**

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b) What is the maximum height of the tree? **2**

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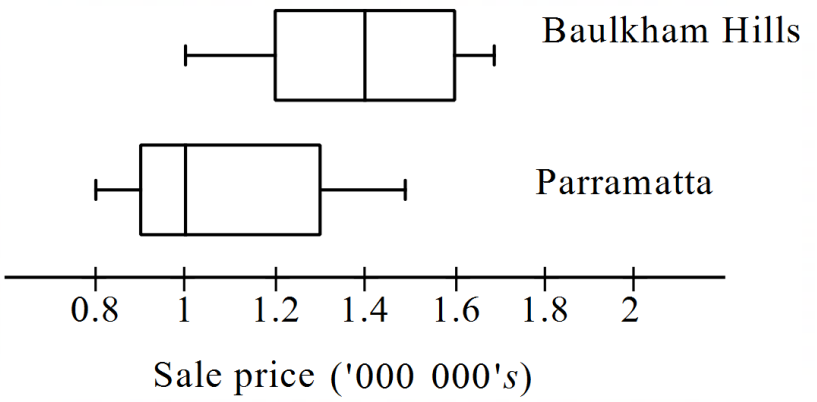
Question 16 (3 marks)

The box and whisker plot below compares sale prices for houses sold in Parramatta and Baulkham Hills in millions of dollars. 120 houses were sold in Baulkham Hills and 24 houses were sold in Parramatta.



a) How many houses in Parramatta were sold for less than the minimum sale price in Baulkham Hills? **1**

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b) Tharuki claims that the top 25% of house sale prices in Baulkham Hills were all above the highest house sale price for Parramatta. Is Tharuki correct? Justify your response, referring to the house sale prices for both suburbs. **2**

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Question 17 (4 marks)

For the arithmetic sequence 507, 501, 495, …, find the:

a) 15^{th} term. **1**

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b) sum of the first 15 terms. **1**

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c) first term that is below 200. **2**

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Question 18 (3 marks)

On Sussana's 21st birthday, she received $5 000 from her parents. This money was placed into a savings account for 5 years at 4% per annum compounded annually. On her 22nd birthday, Sussana deposited $1500 into a savings account which earned 6% per annum. She continued to deposit $1500 every 3 months into this account for 4 years. Sussana withdraws all money from both accounts on her 26th birthday. (Note: Sussana does not make a deposit on her 26th birthday.)

The table below shows the future value of a $1 annuity at different interest rates at different numbers of time periods.

n / i	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%
4	4.06040	4.09090	4.12161	4.15252	4.18363	4.21494	4.24646	4.27819	4.31013	4.34227	4.37462
8	8.28567	8.43284	8.58297	8.73612	8.89234	9.05169	9.21423	9.38001	9.54911	9.72157	9.89747
12	12.68250	13.04121	13.41209	13.79555	14.19203	14.60196	15.02581	15.46403	15.91713	16.38559	16.86994
16	17.25786	17.93237	18.63929	19.38022	20.15688	20.97103	21.82453	22.71934	23.65749	24.64114	25.67253
20	22.01900	23.12367	24.29737	25.54466	26.87037	28.27968	29.77808	31.37142	33.06595	34.86832	36.78559
24	26.97346	28.63352	30.42186	32.34904	34.42647	36.66653	39.08260	41.68920	44.50200	47.53800	50.81558
28	32.12910	34.48148	37.05121	39.85980	42.93092	46.29063	49.96758	53.99333	58.40258	63.23351	68.52811
32	37.49407	40.68829	44.22703	48.15028	52.50276	57.33450	62.70147	68.66625	75.29883	82.67750	90.88978
36	43.07688	47.27597	51.99437	57.30141	63.27594	70.00760	77.59831	86.16397	95.83632	106.76519	119.12087
40	48.88637	54.26789	60.40198	67.40255	75.40126	84.55028	95.02552	107.03032	120.79977	136.60561	154.76197
44	54.93176	61.68887	69.50266	78.55232	89.04841	101.23833	115.41288	131.91384	151.14301	173.57267	199.75803
48	61.22261	69.56522	79.35352	90.85958	104.40840	120.38826	139.26321	161.58790	188.02539	219.36837	256.56453

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How much interest did Sussana earn at the end of the 5 years?

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Question 19 (6 marks)

The following data compares the home loan interest rate (R) and the median house price (P) for Australia.

Interest Rate (%) **(*R*)**	13.5	2.75	7	3	9.5	7	5	4	13
Median House Price ('000's) **(*P*)**	82	500	480	680	82	300	500	580	80

a) The y intercept of the least squares regression line is 748.98 (2 decimal places). Find the equation of the least squares regression line. Give your answer correct to 2 decimal places. **1**

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b) Find Pearson's Correlation Coefficient (r). Give your answer correct to 2 decimal places. **1**

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c) Describe the strength and direction of the correlation between interest rates and the median house price. **2**

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d) Predict the median house price when interest rates are at 10%. Give your answer to the nearest thousand. **2**

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Question 20 (5 marks)

The amount of Cesium-137 (A) after t years can be found using the formula $A = A_0 e^{kt}$.
Initially there are 50mg of Cesium-137 in a water supply. After 30 years, this amount is halved.

a) Show that $k = \frac{1}{30}\ln(0.5)$ **1**

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b) How long will it take for 10mg of Cesium-137 to remain? Give your answer correct to 2 decimal places. **2**

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c) What is the rate of decrease of Cesium-137 after 20 years? Give your answer correct to 2 decimal places. **2**

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Question 21 (6 marks)

Find the following indefinite integrals:

a) $\int (6x^2 + \pi x - 5)dx$ **2**

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b) $\int 8^{(5x-3)}dx$ **2**

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c) $\int \frac{20x}{5x^2+3}dx$ **2**

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Question 22 (4 marks)

A machine fills bags with lollies. The weight of the bag of lollies is normally distributed with a mean of 250g and standard deviation of 2g.

a) Bags with a weight less than 244g are donated to a local charity. According to the Empirical Rule, what percentage of bags produced are donated to charity? **2**

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b) Bags of lollies to be sold are most profitable when their weight is between 245g and 253g. What percentage of the bags of lollies are expected to be most profitable? You may refer to the *z* table at the back of this book. **2**

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Question 23 (6 marks)

A particle moves in a straight line such that its displacement x (metres) from a fixed point O is given by $x = \frac{2t^3}{3} - \frac{t^2}{2} - 15t + 3$, where t is the time in seconds.

a) Find when the particle comes to rest. **2**

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b) Find the acceleration of the particle after 2 seconds. **2**

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c) Find the total distance travelled by the particle in the first 5 seconds. **2**

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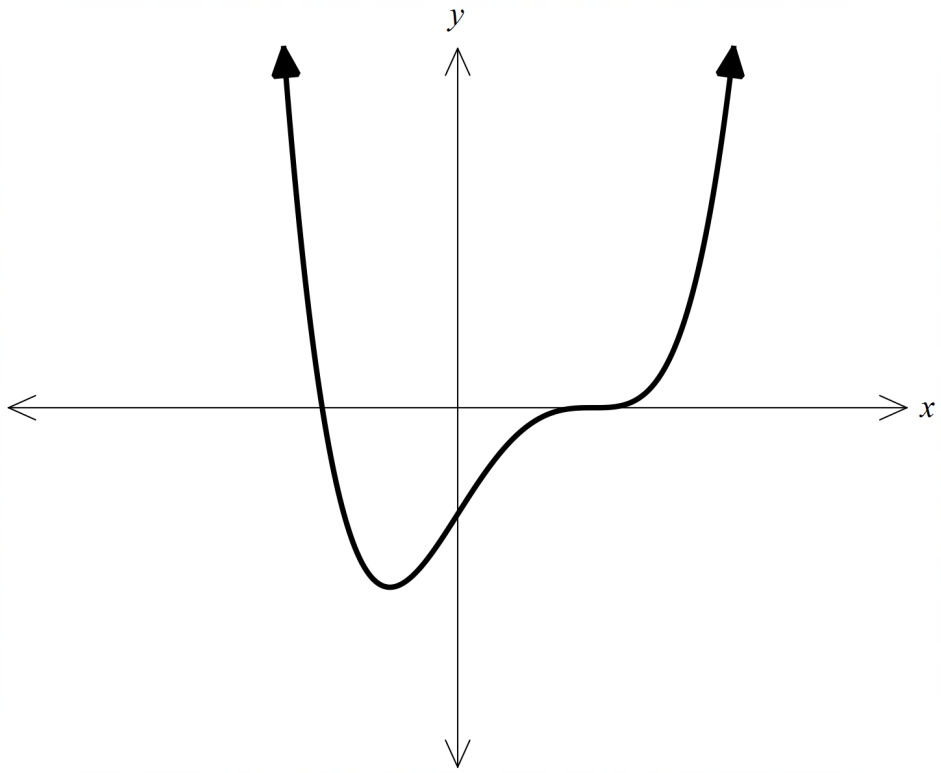
Question 24 (2 marks)

The diagram below shows the graph of $y = f(x)$.



On the same diagram, draw the graph of $y = f'(x)$. **2**

Question 25 (2 marks)

The time (T) taken to fill a swimming pool (in hours) is inversely proportional to the number of hoses (N) filling it. It takes 8 hours to fill a pool using 3 hoses. How long would it take to fill the same pool using 5 hoses? Assume the flow rate of all of the hoses is the same. **2**

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Question 26 (2 marks)

For what values does $kx^2 - 2x + 2 = 0$ have no real roots? **2**

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Question 27 (3 marks)

The wait time, in minutes (x), to speak to an operator in a call centre, is modelled by the following probability density function:

$$f(x) = \begin{cases} \frac{-3}{650}(x+1)(x-10) & 0 \le x \le 10 \\ 0 & \text{otherwise} \end{cases}$$

a) find the mode. **1**

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b) find the probability that a person will need to wait less than 5 minutes before their call is answered. **2**

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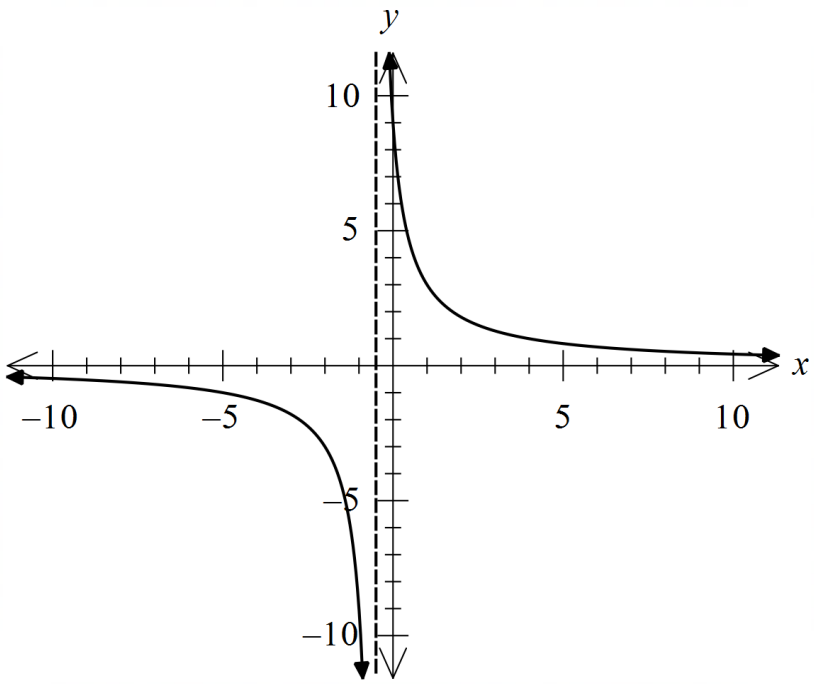
Question 28 (5 marks)

The graph below shows the equation of $y = \dfrac{9}{2x+1}$.



a) Solve $|x+2| = \dfrac{5}{2}$. **2**

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b) Hence or otherwise, on the axis above, graph the equation $y = 5 - 2|x+2|$ showing all essential features. **2**

c) The point of intersection of $y = \dfrac{9}{2x+1}$ and $y = 5 - 2|x+2|$ is $(-5,-1)$. Hence or otherwise solve $\dfrac{9}{2x+1} < 5 - 2|x+2|$. **1**

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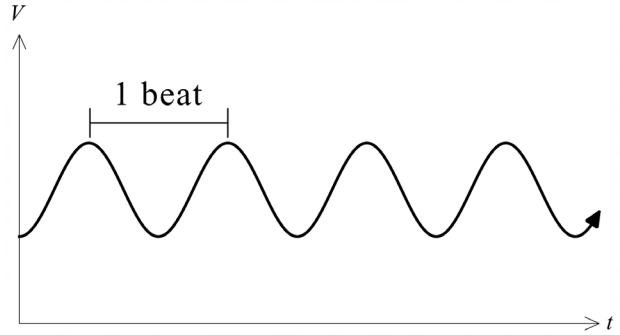
Question 29 (5 marks)

Blood is pumped into and out of a human heart as it beats. The maximum capacity of the average human heart is 140 millilitres and drops to a minimum of 70 millilitres, before refilling again to the maximum of 140 millilitres. The capacity $V(t)$ in millilitres of blood in the heart as a function of time, t seconds, can be modelled by the equation $V(t) = A - B\cos(nt)$. Some of this information is shown in the diagram below.



A person's heart is measured as 60 beats/minute. Initially, their heart has 70 millilitres of blood in it.

When is the first time the volume of the heart will be increasing at 200 millilitres/second? **5**

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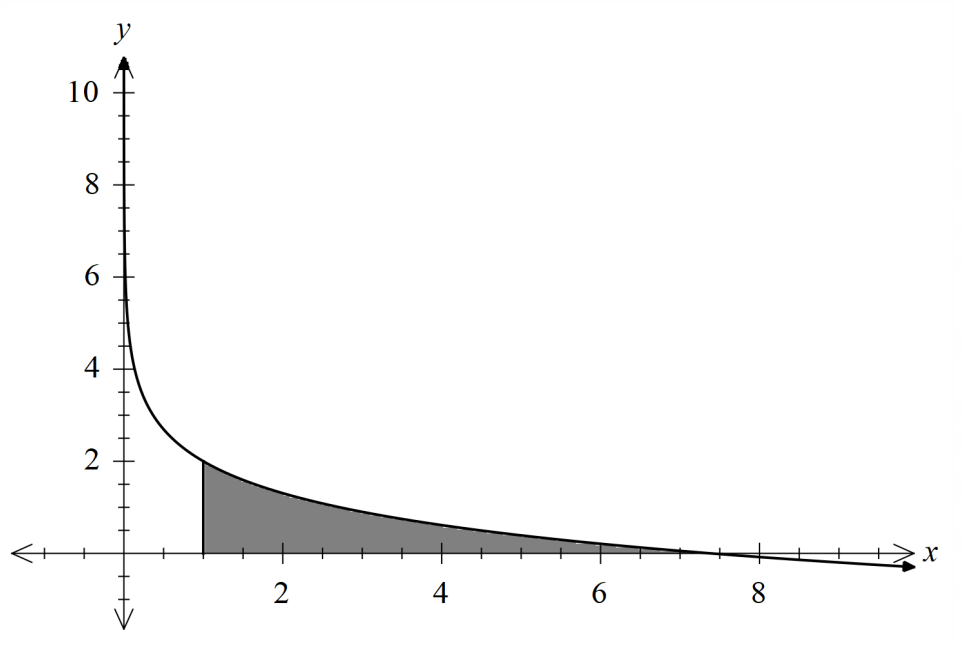
More working out space on the next page

Question 30 (3 marks)

The diagram below shows the graph of the function $x = e^{(2-y)}$.



Find the exact area bounded by the curve, the x-axis and the line $x = 1$ **3**

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Question 31 (4 marks)

a) Differentiate $y = x\cos x$. **2**

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b) Hence or otherwise, evaluate $\int_0^{\pi} x\sin x dx$ **2**

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Question 32 (5 marks)

Aishwarya takes out a home loan for \$750 000 at a rate of 6% pa compounded monthly. At the end of each month, she makes a repayment of \$$M$. She continues making the same monthly repayment \$$M$ until the loan is paid in full after 25 years.

a) Let A_n be the amount in the account after n months. Show that **1**
$A_2 = 750000(1.005)^2 - 1.005M - M$.

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b) Hence, find the monthly repayment. **2**

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Question 32 continues on the following page

c) Aishwarya decides to instead make monthly repayments of $5200. For how many months will Aishwarya need to make full monthly repayments of $5200? **2**

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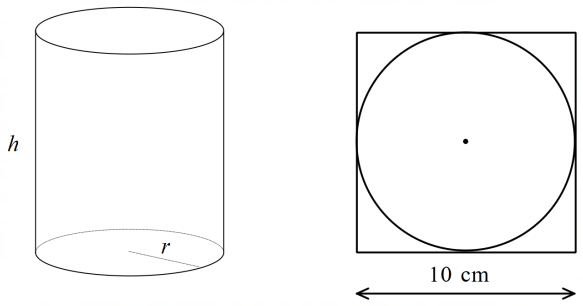
Question 33 (5 marks)

A large cylindrical can with volume 840 cm^3 is to be constructed from 3 sheets of metal. One of the circular ends is cut from a square sheet of metal of side length 10 cm. The other circular end is cut from a second square sheet of metal of side length 90 cm.

To be able to fit items in the cylinder, the radius of the circular ends must be at least 3 cm.



Let the height of the cylinder be h and the radius of each of the circular ends be r.

a) Show that the surface area (A) of the cylinder is given by $A = 2\pi r^2 + \frac{1680}{r}$. **1**

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b) Hence find the minimum surface area of the cylinder to the nearest whole number. **4**

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End of Exam

Baulkham Hills High School

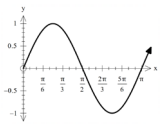
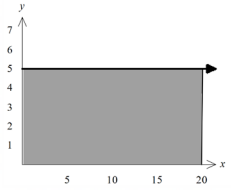
Task 4 Trials Examination 2023

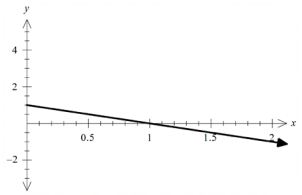
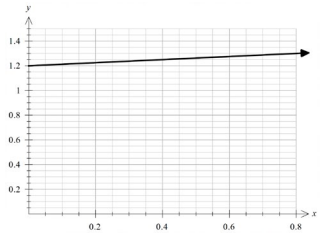
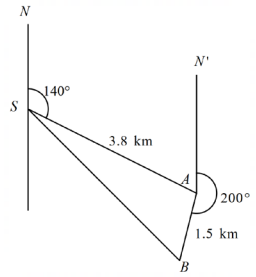
Marking Guideline – Yr 12 Mathematics Advanced

Section I (10 marks)

Award 1 mark to each correct answer.

Answers: 1C 2C 3A 4C 5D 6C 7C 8A 9B 10D

Q	Suggested solutions	A
1	Break even point occurs when costs = income $200+2x=4x$ $2x=200$ $x=100$	C
2		C
3	$E(X)=0\times\frac{1}{8}+1\times\frac{3}{8}+2\times\frac{3}{8}+3\times\frac{1}{8}$ $=1.5$ Alternatively since the probability distribution is symmetrical, expected value must be in the middle.	A
4	At the point $x=a$, the gradient of the tangent is negative ie $f'(A)<0$. At the point $x=a$, the function is concave up ie $f''(A)>0$.	C
5	For a probability density function to exist $f(x)>0$ for all x in the domain and $\int_a^b f(x)dx=1$. A. Since $\sin 2x<0$ when $\frac{\pi}{2}<x<\pi$ (see graph below), $\therefore$ not a PDF  B. $\int_0^{20} 5dx=100$ $\therefore$ not a PDF  C. $f(x)<0$ when $1<x<2$	D

	$\therefore$ not a PDF  D. $f(x) > 0$ for all x $\int_0^{0.8} 1.2 + 0.125x\,dx = \frac{0.8}{2}(1.2 + (1.2 + 0.125 \times 0.8))$ $= 1$ $\therefore$ is a PDF 	
6	 $\angle SAN + \angle NSA = 180°$ (cointerior angles, $NS \parallel N'A$) $\angle SAN' = 40°$ $\angle BAS = 120°$ $SB^2 = 3.8^2 + 1.5^2 - 2 \times 3.8 \times 1.5 \cos 120°$ $= 22.39$ $SB = \sqrt{22.39}$ $= 4.731807266$ $= 4.73 (2dp)$	C

7	$E(X) = \int xf(x)\,dx$ $= 3\int_0^1 x(1-x)^2\,dx$ $= 3\int_0^1 x(1-2x+x^2)$ Var(X) = $= \int_0^1 3(x\ -2x^2+x^3)$ $= \int_0^1 3x^3-6x^2+3x\ \ dx$ $= \left[\frac{3x^4}{4}-2x^3+\frac{3x^2}{2}\right]_0^1$ $= 0.25$	C
8	Domain: $x^2-1\neq 0$ $x\neq \pm 1$ Note: When writing a domain, it should be stated what values of x are included, not just the ones that are excluded. Range:	A

$y = \dfrac{x^2+1}{x^2-1}$ is an even function and so has line symmetry about the y axis.

$$\lim_{x\to\infty}\frac{x^2+1}{x^2-1} = \lim_{x\to\infty}\frac{x^2/\left(x^2+1/x^2\right)}{x^2/\left(x^2-1/x^2\right)}$$

$$= \lim_{x\to\infty}\frac{1+1/x^2}{1-1/x^2}$$

$$= 1$$

$$\lim_{x\to-\infty}\frac{x^2+1}{x^2-1} = 1 \left(\text{since } y = \frac{x^2+1}{x^2-1} \text{is an even function}\right)$$

Cuts the y axis when $x = 0$

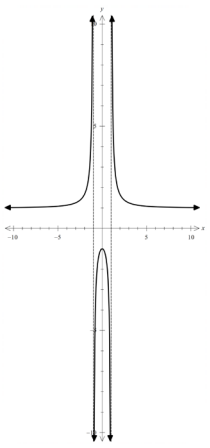
ie when $y = -1$

Cuts the x axis when $y = 0$

no solutions $\rightarrow$ doesn't have x intercepts

$$\lim_{x\to1^+}\frac{x^2+1}{x^2-1} = \infty$$

$$\lim_{x\to1^-}\frac{x^2+1}{x^2-1} = -\infty$$



$\therefore$ Domain: $x \in (-\infty,-1)\cup(-1,1)\cup(1,\infty)$

Range: $y \in (-\infty,-1)\cup(1,\infty)$

9	$g(x) = 2f(1-2x)$ $= 2f(-2(x-\frac{1}{2})$ $f(-2x)$ stretches horizontally by a factor of $\frac{1}{2}$ and reflects in the y ie $(2,-2)$ maps to (-1, -2) $f(-2(x-\frac{1}{2})$ will then translate this function $\frac{1}{2}$ unit to the right. ie the point is now mapped to $(\frac{-1}{2},-2)$ $2f(-2(x-\frac{1}{2})$ stretches vertically by a factor of 2 ie the point now maps to $(\frac{-1}{2},-4)$	B
10	When $-2 \le g(x) \le 2$, then $f(g(x))$ $\therefore$ the domain is $-2 \le x \le 2$	D

Section II (90 marks)

In all questions, award full marks for correct answers with necessary working.

Use the suggested solutions in conjunction with the marking criteria.

Q	Suggested Solutions	Marking Criteria
11	From graph 15% accept 13 – 17% 1 – correct solution	
12a	$y = 5x^4 - 14x^2 + 3$ $\frac{dy}{dx} = 20x^3 - 28x$ 1 – correct solution	
12b	$y = (8-5x)^4$ $\frac{dy}{dx} = 4(8-5x)^3 \times (-5)$ $= -20(8-5x)^3$ 2 – correct solution 1 - obtains $4(8-5x)^3$	
12c	$y = x^2 \ln(3x+5)$ $\frac{dy}{dx} = x^2 \times (\frac{3}{3x+5}) + \ln(3x+5) \times 2x$ $= \frac{3x^2}{3x+5} + 2x\ln(3x+5)$ $= x\left(\frac{3x}{3x+5} + 2\ln(3x+5)\right)$ 2 – correct solution 1 – derives $\ln(3x+5)$ Do not deduct marks for not factorising final answer.	

12d	$y = \dfrac{e^x}{10x-1}$ $\dfrac{dy}{dx} = \dfrac{(10x-1)e^x - e^x(10)}{(10x-1)^2}$ $= \dfrac{e^x(10x-1-10)}{(10x-1)^2}$ $= \dfrac{e^x(10x-11)}{(10x-1)^2}$ $= \dfrac{10xe^x - 11e^x}{(10x-1)^2}$ 2 – correct solution 1 –correctly obtains either numerator or denominator. Do not deduct marks for not factorising final answer.
13a	$A = \dfrac{b-a}{2n}\{f(a)+f(b)+2[f(x_1)+f(x_2)]\}$ $= \dfrac{10}{2}\{20+0+2[25+22]\}$ $= 570m^2$ 2 – correct solution 1 – 1 error in substitution of trapezoidal rule.
13b	As the shape of the land is concave down, the trapezoidal rule will give an underestimate. 1 – correct solution
14	$f'(x) = 8x^3 - 3x^2$ $f(x) = 2x^4 - x^3 + c$ $f(2) = -1$ $2(2)^4 - 2^3 + c = -1$ $c = -25$ $f(x) = 2x^4 - x^3 - 25$ 2 – correct solution 1 – correctly integrating but omits constant or attempts to integrate and correctly finds a constant from incorrect working

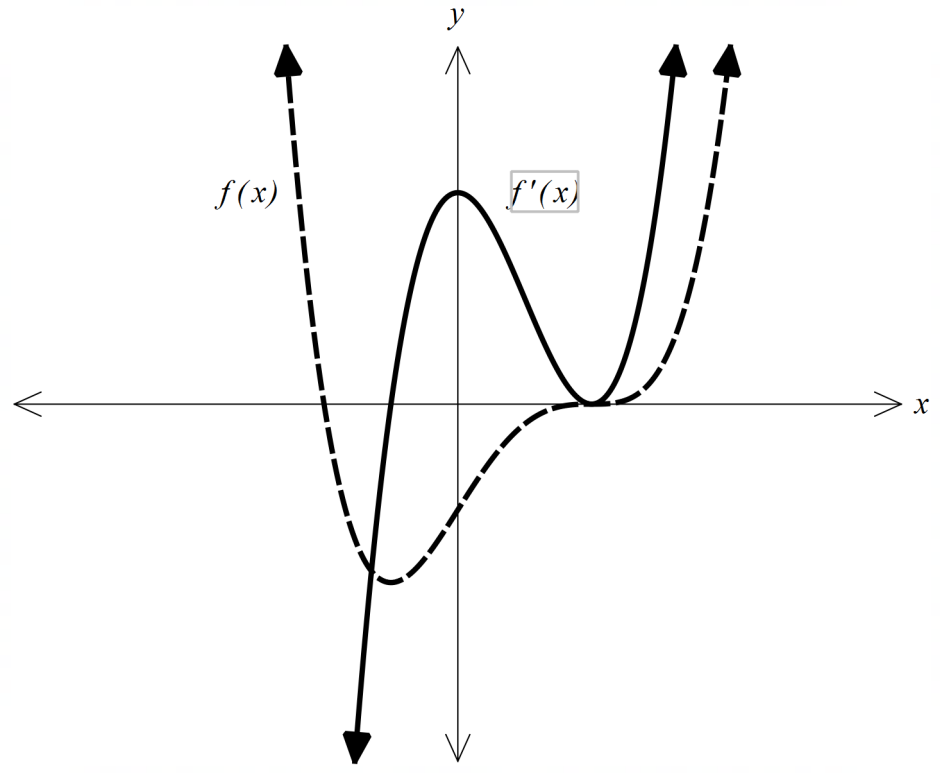
15a	$\text{Week } 1 = 20\% \times 50$ $= 10cm$ $\text{Week } 2 = 20\% \times 10$ $= 2cm$ Growth in $\text{Week } 3 = 20\% \times 2$ $= 0.4cm$ GP where $a = 10, r = 0.2, T_n = ar^{n-1}$ $\text{Growth in Week } 5 = 10 \times 0.2^{5-1}$ $= 0.016cm$ 2-correct solution 1 – obtains $T_n = 10 \times 0.2^{n-1}$ or calculates the growth in week 3 to be 0.4 cm
15b	$S_\infty = \frac{a}{1-r}$ $= \frac{10}{1-0.2}$ $= 12.5$ $\therefore$ Maximum height of the tree is 50 + 12.5 = 62.5cm 2 – correct solution 1 – correctly finding the limiting sum
16a	The median of Parramatta is the same as the lowest price in Baulkham Hills. $\therefore$ 50% of houses sold in Parramatta were below $1 000 000 12 houses 1 – correct solution
16b	The top 25% of houses in Baulkham Hills were sold for at least 1.6 million (Q_3). The highest house sale in Parramatta was 1.5 million. $\therefore$ Tharuki is correct 2 – correct solution 1 – states that the upper quartile for Baulkham Hills is higher than the highest sale in Parramatta.
17a	$T_n = a + (n-1)d$ $T_{15} = 507 + (15-1) \times (-6)$ $= 423$ 1 – correct solution

17b	$S_n = \frac{n}{2}(2a + (n-1)d)$ $= \frac{15}{2}(2 \times 507 + (15-1) \times (-6)$ $= 6975$ 1 – correct solution
17c	$507 - 6(n-1) < 200$ $507 - 6n + 6 < 200$ $-6n < -313$ $n > 52\frac{1}{6}$ $\therefore$ The 53rd term is the first term below 200 $T_{53} = 507 - 6(53) + 6$ $= 195$ 2 - correct solution 1 – correctly solving the inequation
18	Total of investment = $5000(1+0.04)^5 + 1500 \times 17.93237$ $= \$32981.82$ Total Interest $= 32981.82 - 5000 - 1500 \times 4 \times 4$ $= \$3981.82$ 3 - correct solution 2 – correctly calculates future value and annuity OR correctly calculates one of annuity or future value and total interest 1 – correctly calculates one of compound interest or annuity
19a	By calculator, gradient of the line is -53.39 (2dp) $\therefore$ Equation of the line $P = 749.98 - 53.39R$ 1 – correct solution with equation in terms of P and R
19b	By calculator: $r = -0.9211732195$ $= -0.92(2dp)$ 1 – correct solution
19c	Strong, negative correlation 2 – correct solution 1 - Stating either strong or negative

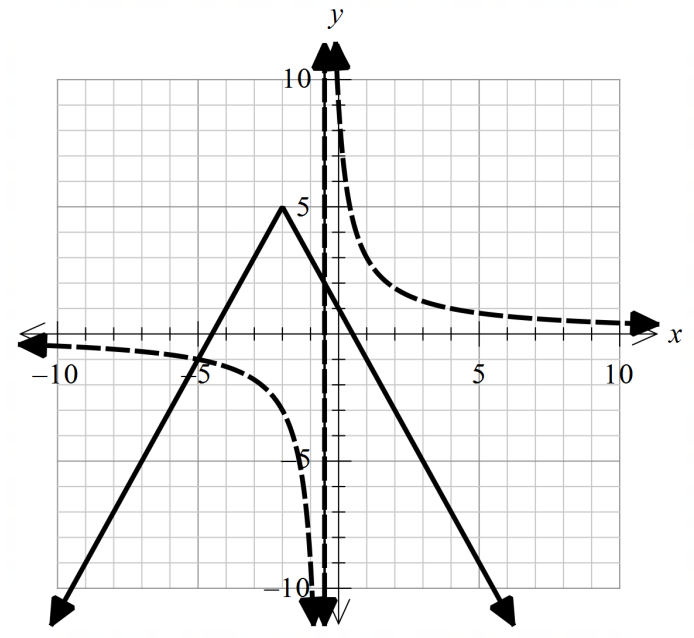
19d	$P = 749.98 - 53.39R$ when $R = 10$, $P = 749.98 - 53.39 \times 10$ $= 215.1088061(thousand)$ $= 215000(nearest\ thousand)$ 2 – correct solution 1 – substituting $R = 10$
20a	$A = A_0 e^{kt}$ when $A_0 = 50, t = 30, A = 25$ $25 = 50e^{30k}$ $0.5 = e^{30k}$ $\ln(0.5) = 30k$ $k = \frac{1}{30}\ln(0.5)$ 1 – correct solution
20b	$10 = 50e^{kt}$ $\frac{10}{50} = e^{kt}$ $\ln\left(\frac{1}{5}\right) = \left(\frac{1}{30}\ln\left(\frac{1}{2}\right)\right)t$ $t = \frac{30\ln 1/5}{\ln 1/2}$ $= 69.65784285$ $= 69.66(2\,dp)$ 2 – correct solution 1 – substitution into formula

20c	$\frac{dA}{dt} = 50ke^{kt}$ when $t = 20$, $\frac{dA}{dt} = 50ke^{20k}$ $= 50 \times \frac{1}{30} \ln\left(\frac{1}{2}\right) e^{20 \times \frac{1}{30} \ln\left(\frac{1}{2}\right)}$ $= -0.7277589362$ $= -0.73 mg / year$ 2 – correct solution 1 – correctly finds $\frac{dA}{dt}$
21a	$\int (6x^2 + \pi x - 5)dx = 2x^3 + \frac{\pi x^2}{2} - 5x + c$ 2 – correct solution including constant 1 – Integrating correctly without constant or attempts to integrate with constant.
21b	$\int 8^{5x-3} dx = \frac{8^{5x-3}}{5 \ln 8} + c$ 2 – correct solution (ignore constant) 1 – obtains $\frac{k \times 8^{5x-3}}{\ln 8}$ or $\frac{8^{5x-3}}{5}$
21c	$\int \frac{20x}{5x^2 + 3} dx = 2\int \frac{10x}{5x^2 + 3} dx$ $= 2 \ln(5x^2 + 3) + c$ 2 – correct solution 1 – $\frac{1}{2} \ln(5x^2 + 3)$ (ignore constant)
22a	$z = \frac{x - \mu}{\sigma}$ $= \frac{244 - 250}{2}$ $= -3$ $\frac{100\% - 99.7\%}{2} = 0.15\%$ 2 – correct solution 1 – correctly calculates z score

22b	When $x = 245$, $z = \frac{245 - 250}{2}$ $= -2.5$ When $x = 253$, $z = \frac{253 - 250}{2}$ $= 1.5$ $P(-2.5 < Z < 1.5) = 0.93319 - (1 - 0.99379)$ $= 0.9268$ $= 92.68\%$ 2 – correct solution 1 - Correctly calculates both z scores and reads one value from table correctly
23a	Particle comes to rest when $v = 0$ $ie\, 2t^2 - t - 15 = 0$ $(2t + 5)(t - 3) = 0$ $t = \frac{-2}{5} or 3$ $t = 3 \,\text{only since}\, t > 0$ 2 – correct solution 1 – differentiates and equates the velocity to 0
23b	$x = \frac{2t^3}{3} - \frac{t^2}{2} - 15t + 3$ $v = \frac{dx}{dt} = 2t^2 - t - 15$ $a = \frac{d^2x}{dt^2} = 4t - 1$ $when\, t = 2, a = 4 \times 2 - 1$ $= 7m/s^2$ 2 – correct solution 1-correctly finds acceleration or correctly finds acceleration from an incorrect velocity equation.

23c	when $t = 0, x = 3$ when $t = 3, x = \frac{2}{3}(3)^3 - \frac{3^2}{2} - 15 \times 3 + 3$ $= -28.5$ When when $t = 5, x = \frac{2}{3}(5)^3 - \frac{5^2}{2} - 15 \times 5 + 3$ $= -\frac{7}{6}$ $\therefore$ Total distance travelled $= 3 - (-28.5) + (28.5 - \frac{7}{6})$ $= 58\frac{5}{6} units$ 2 – correct solution 1 – finds the difference when $x = 5$ and $x = 0$ without considering the particle coming to rest.
24	 2 – correct solution 1 – correct x intercepts

25	$T \alpha \frac{1}{N}$ $T = \frac{k}{N}$ when $T = 8, N = 3$ $8 = \frac{k}{3}$ $k = 24$ $\therefore T = \frac{24}{N}$ $T = \frac{24}{5}$ $= 4.8$ (or 4 hours, 48 minutes) 2 – correct solution 1 – correctly finds k
26	$kx^2 - 2x + 2 = 0$ has no real roots when $\Delta < 0$ ie $2^2 - 4 \times k \times 2 < 0$ $4 - 8k < 0$ $8k > 4$ $k > \frac{1}{2}$ 2 – correct solution 1 – correctly substitutes into the discriminant and puts less than zero.
27a	$Mode = \frac{-1+10}{2}$ since distribution is symmetrical $= 4.5$ 1 – correct solution

27b	$\int_0^5 \frac{-3}{650}(x+1)(x-10)dx = \frac{-3}{650}\int_0^5 (x+1)(x-10)dx$ $= \frac{-3}{650}\int_0^5 x^2 - 9x - 10\,dx$ $= \frac{-3}{650}\left[\frac{x^3}{3} - \frac{9x^2}{2} - 10x\right]_0^5$ $= \frac{-3}{650}\left[\frac{5^3}{3} - \frac{9(5)^2}{2} - 10\times 5\right]$ $= \frac{29}{52}$ 2 – correct solution 1 – attempts to perform definite integral
28a	$\lvert x+2\rvert = \frac{5}{2}$ $x+2 = \frac{5}{2}$ or $x+2 = -\frac{5}{2}$ $x = \frac{1}{2}$ or $x = -4\frac{1}{2}$ 2 – correct solution 1 – one solution
28b	 2 – correct solution 1 – vertex + y intercept labelled

28c	$-5 < x < -\frac{1}{2}$ 1 – correct solution
29	140mL is the maximum amount of blood in the heart & 70mL is the minimum amount of blood at any one time. $\therefore$ Amplitude is 35 ie B = 35 $Mean\,Value = \frac{140+70}{2}$ $= 105$ $\therefore A = 105$ Since athlete's heart beats at 60 beats/minute $\rightarrow$ 1 beat/second or 1 second/beat *ie* Period =1 $\frac{2\pi}{n} = 1$ $n = 2\pi$ $V(t) = 105 - 35\cos(2\pi t)$ $V'(t) = 35 \times 2\pi \sin(2\pi t)$ $200 = 70\pi \sin(2\pi t)$ $\sin(2\pi t) = \frac{200}{70\pi}$ $2\pi t = 1.14197583$ $t = 0.1817510982$ *ie* after 0.18 seconds (2 dp) 5 – correct solution 4 – obtains $\sin(2\pi t) = \frac{200}{70\pi}$ 3 – correctly differentiates $V(t)$ 2 – Finds n, A and B 1 – Finds A and B or n or obtains $V'(t) = Bn \sin(nt)$
30	when $x = 1, y = 2$. $Total\ Area = \int_0^2 e^{2-y}dy - 2 \times 1$ $= \left[-e^{2-y}\right]_0^2 - 2$ $= -1 + e^2 - 2$ $= e^2 - 3$ 3 – correct solution 2 – correctly integrates $\int_0^2 e^{2-y}dy$ 1 – finds when $x = 1, y = 2$ or obtains $\int_1^{e^2} 2 - \ln x\,dx$

31a	$y = x\cos x$ $\frac{dy}{dx} = \cos x - x\sin x$ 2 – correct solution 1 – attempts to use product rule
31b	$\int_0^{\pi} (\cos x - x\sin x)dx = \left[x\cos x\right]_0^{\pi}$ $\int_0^{\pi} \cos x dx - \int_0^{\pi} x\sin x dx = \left[x\cos x\right]_0^{\pi}$ $\int_0^{\pi} x\sin x = \int_0^{\pi} \cos x dx - \left[x\cos x\right]_0^{\pi}$ $= \left[\sin x - x\cos x\right]_0^{\pi}$ $= \sin\pi - \pi\cos\pi - (0-0)$ $= \pi$ 2 – correct solution 1 – correctly integrates $\sin x$
32a	Let A_n be the amount in the account after n months $A_1 = 750000(1.005) - M$ $A_2 = A_1(1.005) - M$ $= (750000(1.005) - M)(1.005) - M$ $= 750000(1.005)^2 - 1.005M - M$ 1 - correct solution

32b	$A_3 = A_2(1.005) - M$ $= (750000(1.005)^2 - 1.005M - M)(1.005) - M$ $= 750000(1.005)^3 - 1.005^2 M - 1.005M - M$ ..... $A_{300} = 750000(1.005)^{300} - 1.005^{299} M - 1.005^{298} M - .... - 1.005M - 1$ $= 750000(1.005)^{300} - M(1 + 1.005 + 1.005^2 + 1.005^3 + ... + 1.005^{299})$ $= 750000(1.005)^{300} - M(\frac{1.005^{300} - 1}{1.005 - 1})$ *But* $A_{300} = 0$ $0 = 750000(1.005)^{300} - M(\frac{1.005^{300} - 1}{1.005 - 1})$ $M(\frac{1.005^{300} - 1}{0.005}) = 750000(1.005)^{300}$ $M = \frac{750000(1.005)^{300} \times 0.005}{1.005^{300} - 1}$ $= 4832.260511$ $= 4832.26$ 2 – correct solution 1 – uses the sum of a GP formula to establish an equation for A_{300}

32c	$750000(1.005)^n - 5200(\frac{1.005^n - 1}{1.005 - 1}) = A_n$ (from (a)) $750000(1.005)^n - 5200(\frac{1.005^n - 1}{0.005}) = 0$ $5200(\frac{1.005^n - 1}{0.005}) = 750000(1.005)^n$ $1040000(1.005^n - 1) - 750000(1.005)^n = 0$ $1040000 \times 1.005^n - 1040000 - 750000 \times 1.005^n = 0$ $1.005^n(1040000 - 750000) = 1040000$ $290000(1.005)^n = 1040000$ $1.005^n = \frac{104}{29}$ $\log_{1.005}\left(\frac{104}{29}\right) = n$ $n = 256.0570306$ $= 256$ full payments can be made 2 – correct solution 1 – obtains $1.005^n = \frac{98}{29}$
33a	$V = \pi r^2 h$ $\pi r^2 h = 840$ $h = \frac{840}{\pi r^2}$ $A = 2\pi r^2 + 2\pi rh$ $= 2\pi r^2 + 2\pi r\left(\frac{840}{\pi r^2}\right)$ $= 2\pi r^2 + \left(\frac{1680}{r}\right)$ 1 – correct solution

33b

$$A = 2\pi r^2 + \frac{1680}{r}$$
$$= 2\pi r^2 + 1680r^{-1}$$
$$\frac{dA}{dr} = 4\pi r - 1680r^{-2}$$

Stationery points occur when $\frac{dA}{dr} = 0$

$$4\pi r - 1680r^{-2} = 0$$
$$4\pi r - \frac{1680}{r^2} = 0$$
$$4\pi r = \frac{1680}{r^2}$$
$$4\pi r^3 = 1680$$
$$r^3 = \frac{420}{\pi}$$
$$r = \sqrt[3]{\frac{420}{\pi}}$$
$$r = 5.113282718$$

but $r > 5$

$$\frac{d^2A}{dr^2} = 4\pi + 3360r^{-3}$$
$$= 4\pi + \frac{3360}{r^3}$$

Since $r > 0, \frac{d^2A}{dr^2} > 0$ ie A is always concave up

$\therefore r = 5.11...$ is a minimum turning point

But $3 \le r \le 5$

$\therefore$ Minimum must occur when $r = 5$

When $r = 5, A = 2 \times \pi \times 5^2 + \frac{1680}{5}$
$= 493.0796327\text{cm}$
$\therefore \textit{Minimum Area} = 493\text{cm}\,(\textit{nearest}\ \text{cm})$

4 –correct solution

3 – shows r is a minimum at $r = \sqrt[3]{\frac{420}{\pi}}$ **or** correctly finds the value of r which obtains max/min **and** calculates the area when $r = 5$

2-finds the value of r which obtains max/min

1 – correctly differentiates and equates to 0